# Molecular Interaction of Water Vapor and Oxygen 

Russell G. Wylie* and Robert S. Fisher<br>CSIRO Division of Applied Physics, National Measurement Laboratory, Lindfield, Australia 2070


#### Abstract

The factor $f_{w}(T, p)$ by which oxygen increases the saturation vapor pressure of water has been measured accurately for temperatures of 25,50 , and $75{ }^{\circ} \mathrm{C}$ and pressures to 14 MPa . This paper outlines the measurements and describes their analysis to give the interaction second virial coefficient $B_{o w}(T)$ and the interaction third virial coefficient $\mathrm{C}_{\text {oow }}(\mathrm{T})$ of the vapor - gas mixture. Because the work largely parallels the authors' work for water in air, already reported in detail, the present paper is relatively brief. Fitting Lennard-J ones potentials to the values of $B_{\text {ow }}(T)$ for the three temperatures shows the values to be highly mutually consistent and allows this coefficient and its derivatives to be cal culated over a widetemperature range. The three values of $\mathrm{C}_{\text {oow }}(\mathrm{T})$ define the magnitude of this quantity well for temperatures from about 0 to $100^{\circ} \mathrm{C}$, but for lack of a reliable theory of the form of its temperature dependence yield little information concerning its derivatives. No earlier published measurements or theoretical values of the coefficients appear to exist. The work allows the vapor-pressure enhancement factor $f_{w}(T, p)$ to be calculated for a wide range of conditions and provides a basis for the calculation of the thermodynamic properties of the water-oxygen system.


## Introduction

The effect of oxygen on the concentration of water vapor in equilibrium with liquid water or ice is a little less than that of air. For example, for $20^{\circ} \mathrm{C}$ and a pressure of 15 MPa the approximate effect of the intermolecular forces in the gas phase is an increase of $34 \%$ (for air, $39 \%$ ), that of the pressure acting on the liquid phase (the Poynting effect) an increase of $12 \%$ (12\%), and that of the oxygen dissol ved in the liquid (the Raoult effect) a decrease of $0.3 \%$ (0.2\%). The net effect is an increase of approximately 49\% (55\%).

As for water in air (Wylie and Fisher, 1996), we define the vapor-concentration enhancement factor $g_{w}(T, p)$ by

$$
\begin{equation*}
c_{w}=g_{w}(T, p) c_{w}^{o} \tag{1}
\end{equation*}
$$

where $c_{w}$ is the saturation concentration in moles per unit volume of water vapor in oxygen at the temperature $T$ and total pressure $p$ and $c_{w}^{0}$ that of the pure vapor at the same temperature. Remembering that the partial pressure $p_{w}$ of the vapor is the product of the vapor mole fraction $x_{w}$ and the total pressure, we also define a vapor-pressure enhancement factor $f_{w}(T, p)$ by

$$
\begin{equation*}
p_{w}=f_{w}(T, p) p_{w}^{o} \tag{2}
\end{equation*}
$$

where $p_{w}^{o}$ is the saturation vapor pressure of pure water at $T$. Equation 2 is equivalent to

$$
\begin{equation*}
x_{w}=f_{w} p_{w}^{o} / p \tag{3}
\end{equation*}
$$

It is easily shown that

$$
\begin{equation*}
f_{w}=\left(Z / Z_{w}^{0}\right) g_{w} \tag{4}
\end{equation*}
$$

where $Z_{w}^{0}$ is the compressibility factor of the pure saturated vapor and $Z$ that of the saturated mixture.

Because of their close similarity to those detailed for water in air in our earlier paper (Wylie and Fisher, 1996), the theory, the method of measuring $f_{w}$, and the procedure
for analyzing the experimental results need only be outlined here. However, some of the property data needed for the analysis require detailed consideration.

Throughout the paper the term "uncertainty", unless qualified, means the $3 \sigma$ value obtained by summing the squares of its random and systematic parts. In the terminology developed by the International Organization for Standardization (ISO), the uncertainties quoted are expanded uncertainties obtained by multiplying the appropriately estimated standard deviations by a coverage factor of 3.

## Outline of the Theory

To obtain the equation for $\ln f_{w}$, the thermodynamic potential of the water in the gas mixture is equated to that of the water in the liquid phase. The corresponding equation for pure water vapor in equilibrium with liquid water is then subtracted. The required expression for the thermodynamic potential of the water in the gas is derived using the virial form of the equation of state of the mixture as this has an exact statistical-thermodynamic basis. This equation is

$$
\begin{align*}
& \mathrm{p} \tilde{V} / R T=1+\left(B_{o o} x_{o}^{2}+2 B_{o w} x_{o} x_{w}+B_{w w} x_{w}{ }^{2}\right) \tilde{N}+ \\
& \left(C_{o o o} x_{o}^{3}+3 C_{o o w} x_{o}^{2} x_{w}+3 C_{o w w} x_{o} x_{w}^{2}+C_{w w w} x_{w}^{3}\right) \tilde{N}^{2}+\ldots \tag{5}
\end{align*}
$$

where $x_{0}=1-x_{w}$ is the mole fraction of oxygen, $\tilde{V}$ is the molar volume of the mixture, $B_{o o}$ and $B_{w w}$ are the second virial coefficients of, respectively, oxygen and water vapor, $\mathrm{C}_{000}$ and $\mathrm{C}_{w w w}$ are the corresponding third virial coefficients, and $B_{\text {ow }}, C_{\text {oow }}$, and $C_{\text {oww }}$ are interaction virial coefficients. $B_{\text {ow }}$ represents the interaction of a single oxygen molecule with a single water molecule, $\mathrm{C}_{\text {oow }}$ the interaction of two oxygen molecules with a single water molecule, and $\mathrm{C}_{\text {oww }}$ the interaction of a single oxygen molecule with two water molecules.

When expressions for the terms representing the Poynting and Raoult effects are obtained from simple thermodynamic formulas, the equation for $\ln f_{w}$ is found to be

$$
\begin{equation*}
\ln f_{w}=\ln \left(Z / Z_{w}^{o}\right)-\left(W-W^{0}\right)+(1 / R T) \int_{p_{w}^{o}}^{p} \tilde{V}_{w}^{1, o} d p-x_{o}^{1} \tag{6}
\end{equation*}
$$

where $R$ is the molar gas constant, $\tilde{V}_{w}^{1,0}$ is the molar volume of pure liquid water, and $x_{0}^{\prime}$ is the mole fraction of dissolved oxygen in the liquid, and where

$$
\begin{equation*}
W=2 \frac{\beta}{\tilde{V}}+\frac{3}{2} \frac{\gamma}{\tilde{V}^{2}}+\frac{4}{3} \frac{\delta}{\tilde{v}^{3}}+\ldots \tag{7}
\end{equation*}
$$

in which

$$
\begin{gather*}
\beta=\beta_{o w} x_{o}+\mathrm{B}_{\mathrm{ww}} \mathrm{x}_{\mathrm{w}} \\
\gamma=\mathrm{C}_{\mathrm{oow}} \mathrm{x}_{o}^{2}+2 \mathrm{C}_{\mathrm{oww}} \mathrm{x}_{\mathrm{o}} \mathrm{x}_{\mathrm{w}}+\mathrm{C}_{\mathrm{www}} \mathrm{x}_{\mathrm{w}}{ }^{2} \\
\delta=\mathrm{D}_{\text {ooow }} \mathrm{x}_{\mathrm{o}}^{3}+3 \mathrm{D}_{\mathrm{o} w \mathrm{w}} \mathrm{x}_{\mathrm{o}}^{2} \mathrm{x}_{\mathrm{w}}+3 \mathrm{D}_{\mathrm{owww}} \mathrm{x}_{\mathrm{o}} \mathrm{x}_{\mathrm{w}}{ }^{2}+\mathrm{D}_{\mathrm{wwww}} \mathrm{x}_{\mathrm{w}}{ }^{3} \tag{8}
\end{gather*}
$$

and so on. To obtain $\mathrm{W}^{\circ}$, we simply take $\mathrm{x}_{0}=0$ in eq 8 . In view of eq 4, the expression for $\ln g_{w}$ is just the right-hand side of eq 6 with the first term deleted.

As $f_{w}$ is involved in the calculation of $x_{w}(e q 3)$ which in turn is involved in the calculation of $\tilde{V}$ and $Z$, the righthand side of eq 6 has a small implicit dependence on $f_{w}$. Therefore, to calculate $f_{w}$ from the equation when all the property values are known, an iteration procedure is appropriate. The sensitivites of the calculated $f_{w}$ to changes in the values substituted for $\mathrm{B}_{\text {ow }}, \mathrm{C}_{\text {oow, }}$, and $\mathrm{C}_{\text {oww }}$ are numerically not very different from the corresponding sensitivities for water in air illustrated in Figure 1 of Wylie and Fisher (1996). The use of eq 6 to derive data for $\mathrm{B}_{\mathrm{ow}}(\mathrm{T})$ and $C_{\text {oow }}(T)$ from measurements of $f_{w}(T, p)$ when the other physical properties are known is outlined below.

## Measurements

Brief Outline of the Method. The measurements of $f_{w}$ were made using the same apparatus as employed for water in air (Wylie and Fisher, 1996). Essentially, a flow of oxygen from high-pressure gas cylinders is reduced in pressure to the desired value and passed through a twostage saturator in a temperature-controlled bath. The vapor-gas mixture is then further reduced to a convenient pressure of approximately 1400 kPa and passed through a train of cells in which the water is extracted chemically for weighing. The then dry gas is collected in a light-weight pressure vessel and also weighed.

The temperature and pressure in the saturator are held precisely steady during a run and measured accurately. The quantities of water and oxygen are determined by accurate weighing with detailed attention to buoyancy effects and to some small corrections necessitated by the procedure. Typically, in a single run approximately 600 g of oxygen is passed at a uniform rate for a period of several hours. Much additional time is involved in preparing the apparatus, setting the conditions, and carrying out the numerous weighing operations involved.

The value of $f_{w}$ is calculated from eq 3 in the more explicit form

$$
\begin{equation*}
f_{w}=\frac{m_{w} / M_{w}}{m_{0} / M_{o}+m_{w} / M_{w}}\left[\frac{p}{p_{w}^{o}(T)}\right] \tag{9}
\end{equation*}
$$

where $M_{w}$ and $m_{w}$ are, respectively, the molecular weight and observed mass of the water and $M_{o}$ and $m_{0}$ the corresponding quantities for the oxygen.

Oxygen. The gas used was prepared commercially by the fractional distillation of liquified air and supplied in

Table 1. Experimental Values for the Vapor-Pressure (or Mole-Fraction) Enhancement Factor $f_{w}$ for Water in Oxygena

| $\mathrm{p} / \mathrm{MPa}$ | $\mathrm{t}=25^{\circ} \mathrm{C}$ | $\mathrm{t}=50^{\circ} \mathrm{C}$ | $\mathrm{t}=75^{\circ} \mathrm{C}$ |
| :---: | :---: | :---: | :---: |
| 2.1580 | 1.0497 | 1.0428 |  |
| 3.5360 | 1.0843 | 1.0683 | 1.0610 |
| 6.9850 | 1.1654 | 1.1360 | 1.1175 |
| 10.1010 | 1.2494 | 1.2002 | 1.1681 |
| 14.0850 | 1.3525 | 1.2833 | 1.2370 |
|  |  |  |  |
| a Each value is the mean of, on the average, five results. |  |  |  |

cylinders at a pressure of approximately 24 MPa . Because of the similarity of the intermolecular forces in oxygen and argon, the latter is by a wide margin the principal impurity in the gas. In air, the quantity of argon expressed as a percentage by volume of the quantity of oxygen is approximately 4.4\%. The supplier regarded the oxygen used as containing $0.5-1.0 \%$ of argon by volume. No determination of the argon content of the oxygen actually used was made, but it is shown below that the absence of this information does not significantly affect the highly accurate data for $\mathrm{B}_{\text {ow }}, \mathrm{C}_{\text {oow }}, \mathrm{f}_{\mathrm{w}}$, and $\mathrm{g}_{\mathrm{w}}$ derived from the measurements. In fact, it will be seen from the results that the argon content could not have varied much from $0.4 \%$ by volume throughout the work.

Results. For temperatures of 25 and $50^{\circ} \mathrm{C}$ measurements of $f_{w}$ were made for five pressures from 2.158 to 14.085 MPa , and for a temperature of $75{ }^{\circ} \mathrm{C}$ for four pressures from 3.536 to 14.085 MPa . In calculating $\mathrm{f}_{\mathrm{w}}$ from the measurements by means of eq 9 , the molecular weights used have been based on ${ }^{12} \mathrm{C}=12$, and that of the oxygen has been the value for the pure gas (31.9988). The implications of the latter choice are considered below. The molecular weight of water is 18.0153 . Its saturation vapor pressure has been given by Wexler (1976).

On the average, five runs were made for each set of conditions. A summary of the results is given in Table 1. For purposes of this summary, each individual result was adjusted to correspond accurately to the nominal experimental conditions, and the mean then calculated for each set. The deviations of the individual results may be seen in Figure 1, referred to further below.

## Derivation of $B_{\text {ow }}(T)$ and $C_{\text {oow }}(T)$

To derive data for $B_{\text {ow }}(T)$ and $C_{o o w}(T)$ from the experimental data for $f_{w}(T, p)$ using eq 6 , we require independent data for $B_{w w}(T), C_{w w w}(T), C_{o w w}(T), \tilde{V}_{w}^{1,0}(T, p)$, and $x_{o}^{1}(T, p)$. Also, the equation of state of oxygen is needed in the calculation of $Z(T, p)$ and hence also $\bar{V}(T, p)$ for the mixture. The appropriate data for $B_{w w}(T), C_{w w w}(T)$, and $\tilde{V}_{w}^{1,0}(T, p)$ are as for water in air (Wylie and Fisher, 1996). For $x_{0}^{\prime}$ ( $T, p$ ), which is only marginally significant, the temperature dependence of the low-pressure solubility of oxygen in water has been taken from Himmel blau (1960), and the pressure dependence of the solubility from Zoss et al. (1954). The generation of data for $\mathrm{C}_{\mathrm{oww}}(\mathrm{T})$ and the treatment of the data available for the equation of state of oxygen warrant detailed consideration.

Estimation of $\mathbf{C}_{\text {oww }}(\mathbf{T})$. For water in air the only data available for the corresponding quantity $\mathrm{C}_{\mathrm{aww}}(\mathrm{T})$ was a theoretical estimate published by Hyland and Mason (1975), which we adopted with a $3 \sigma$ uncertainty of $200 \%$. For $\mathrm{C}_{\text {oww }}(\mathrm{T})$ not even an estimate has been given, and so we have used the Hyland and Mason procedure to make our own. This involves the extension to mixtures of Woolley's model (1953) of an imperfect gas as consisting of a perfect gas containing single molecules (monomers), dimers, trimers, etc., and the treatment (to third virial and


Figure 1. Deviations of the individual experimental values of $f_{w}$ from the values given by eq 6 with $\mathrm{B}_{\mathrm{ow}}(\mathrm{T})$ and $\mathrm{C}_{\text {oow }}(\mathrm{T})$ fitted separately for each temperature (Table 4).
third interaction virial coefficients) of the water-oxygen mixture as consisting of oxygen monomers, water monomers, and water dimers. These species self-interact and mutually interact two by two, except that water monomers do not interact with one another as this would be redundant with the inclusion of the dimers.

Following Hyland and Mason (1975), the required coefficient $\mathrm{C}_{\text {oww }}(\mathrm{T})$ is given by

$$
\begin{equation*}
C_{o w w}=-(2 / 3) B_{o w 2} B_{w w} \tag{10}
\end{equation*}
$$

where w2 denotes the dimer. The experimental quantity $\mathrm{B}_{\mathrm{ww}}$ is the same in our calculation as in that for water in air. To obtain $\mathrm{B}_{\text {ow2 }}$, the well depth $\epsilon$ and distance parameter $\sigma$ for the equivalent Lennard-J ones (L-J) (6-12) intermolecular potential (Hirschfelder et al., 1964) are estimated by applying conventional combining rules to the corresponding parameters for oxygen-oxygen interactions and the spherical parts of the water-water interaction and the water-dimer interaction.

We derive the appropriate parameters for the oxygenoxygen interaction from the experimental data of Michels, Schamp, and de Graaff (1954), finding

$$
\begin{equation*}
\epsilon_{\mathrm{od}} / \mathrm{k}=121 \mathrm{~K} \quad \text { and } \quad \sigma_{o \mathrm{o}}=3.39 \times 10^{-10} \mathrm{~m} \tag{11}
\end{equation*}
$$

where k is the Boltzmann constant. The corresponding parameters for the spherical parts of the water-water and water-dimer interactions are obtained directly from Hy Iand and Mason. For the latter interaction we adopt the values derived from viscosity data. We then obtain

$$
\begin{equation*}
\epsilon_{\mathrm{ow} 2} / \mathrm{k}=327 \mathrm{~K} \text { and } \sigma_{\mathrm{ow} 2}=3.48 \times 10^{-10} \mathrm{~m} \tag{12}
\end{equation*}
$$

from which $B_{o w 2}(T)$ can be calculated.

Table 2. Values of the Interaction Virial Coefficient $\mathrm{C}_{\text {oww }}(\mathrm{T})$ for Water Vapor in Oxygen, Derived Theoretically by the Method of Hyland and Mason (1975)

| $\mathrm{t} /{ }^{\circ} \mathrm{C}$ | $-10^{-3} \mathrm{C}_{\text {oww }}(\mathrm{T}) /\left(\mathrm{cm}^{3} / \mathrm{mol}\right)^{2}$ | $\mathrm{t} /{ }^{\circ} \mathrm{C}$ | $-10^{-3} \mathrm{C}_{\text {oww }}(\mathrm{T}) /\left(\mathrm{cm}^{3} / \mathrm{mol}\right)^{2}$ |
| ---: | :---: | ---: | :---: |
| 0 | 225 | 60 | 61 |
| 10 | 174 | 70 | 52 |
| 20 | 137 | 80 | 44 |
| 30 | 110 | 90 | 37 |
| 40 | 89 | 100 | 32 |
| 50 | 74 |  |  |

Table 3. Skeletal Table of the Compressibility Factor Z(T,p) Adopted for Pure Oxygen

| $\mathrm{t} /{ }^{\circ} \mathrm{C}$ | $\mathrm{p} / \mathrm{MPa}=3$ | $\mathrm{p} / \mathrm{MPa}=6$ | $\mathrm{p} / \mathrm{MPa}=9$ | $\mathrm{p} / \mathrm{MPa}=12$ | $\mathrm{p} / \mathrm{MPa}=15$ |
| ---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0.9723 | 0.9482 | 0.9284 | 0.9139 | 0.9052 |
| 25 | 0.9817 | 0.9665 | 0.9550 | 0.9475 | 0.9443 |
| 50 | 0.9885 | 0.9796 | 0.9737 | 0.9710 | 0.9715 |
| 75 | 0.9933 | 0.9890 | 0.9870 | 0.9877 | 0.9909 |
| 100 | 0.9969 | 0.9958 | 0.9968 | 0.9998 | 1.0051 |

Applying eq 10, we find the values of $\mathrm{C}_{\text {oww }}$ tabulated for temperatures of $0-100{ }^{\circ} \mathrm{C}$ in Table 2. The uncertainties involved are similar to those for water in air, and again we adopt a $3 \sigma$ value of $200 \%$.

Equation of State of Oxygen. Reliable PVT data for oxygen are sparse for temperatures above $0^{\circ} \mathrm{C}$ and the present pressures. For the analysis we must know the compressibility factor $Z=p \tilde{V} / R T$ of oxygen for temperatures of 25,50 , and $75^{\circ} \mathrm{C}$ and pressures to approximately 14 MPa . Further, we wish to extend our calculations of $g_{w}$ and $f_{w}$ to 0 and $100{ }^{\circ} \mathrm{C}$, and to 15 MPa . The most reliable data, those of Michels et al. (1954), are for temperatures of 0,25 , and $50^{\circ} \mathrm{C}$ and pressures to 13 MPa (but 12 MPa at $0^{\circ} \mathrm{C}$ ). We have adopted these, and interpolated and extrapolated them by a special physically-based method. This takes advantage of the close similarity of the equations of state of oxygen and nitrogen and the especially good knowledge which exists of the latter.

Using the principle of corresponding states (Hirschfel der et al., 1964), the data of Michels et al. may be correlated with corresponding data taken from the J acobsen and Stewart collation (1973) of the results of various researchers. By minimizing the rms error of the correlation, the relationship between corresponding temperatures, that between corresponding pressures, and hence that between corresponding molar volumes are found to be

$$
\begin{align*}
& \mathrm{T}_{\mathrm{N}_{2}}=0.80438 \mathrm{~T}_{\mathrm{O}_{2}} \\
& \mathrm{p}_{\mathrm{N}_{2}}=0.64450 \mathrm{p}_{\mathrm{O}_{2}}  \tag{13}\\
& \tilde{\mathrm{~V}}_{\mathrm{N}_{2}}=1.24807 \tilde{\mathrm{~V}}_{\mathrm{O}_{2}}
\end{align*}
$$

Over the range of the Michels data, the rms error of the correlation is only 4.0 parts in $10^{4}$. However, the error is largely systematic. Its trend suggests that a modest reduction in the maximum error in using the correlation to extrapolate the $Z$ for oxygen to $100^{\circ} \mathrm{C}$ and to 15 MPa is likely to be gained if the resulting $Z$ is multiplied by the factor

$$
\begin{equation*}
1+6.0 \times 10^{-4}\left[(\mathrm{p} / \mathrm{MPa}-6.0)^{2} / 36.0-1\right] \tag{14}
\end{equation*}
$$

Some sample values of the $Z$ which result from the whole procedure are given in Table 3. We estimate that for temperatures from 0 to $50^{\circ} \mathrm{C}$ the $3 \sigma$ uncertainty in this Z arises largely from that in the Michels data and increases in proportion to the pressure to 15 parts in $10^{4}$ at 15 MPa . For higher temperatures, up to $100^{\circ} \mathrm{C}$, we estimate that

Table 4. Values of $a, B_{o w}, C_{\text {oow, }}$, and Their $3 \sigma$ Uncertainties for the Experimental Temperatures

| t/ ${ }^{\circ} \mathrm{C}$ | $10^{4} a$ | $\begin{aligned} & 10^{4} \times \text { uncertainty } \\ & \text { rand } \end{aligned}$ | $\mathrm{B}_{\text {ow }} /\left(\mathrm{cm}^{3} / \mathrm{mol}\right)$ | uncertainty/(cm³/mol) |  |  | $\mathrm{Coow}_{\text {/ }}\left(\mathrm{cm}^{3} / \mathrm{mol}\right)^{2}$ | uncertainty/(cm $\left.{ }^{3} / \mathrm{mol}\right)^{2}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | rand | syst | total |  | rand | syst | total |
| 25 | -10 | 35 | -27.04 | 1.38 | 0.45 | 1.44 | 1322 | 270 | 54 | 270 |
| 50 | -9 | 26 | -20.58 | 1.05 | 1.02 | 1.47 | 1121 | 230 | 90 | 240 |
| 75 | -11 | 35 | -15.40 | 1.14 | 1.95 | 2.25 | 1060 | 230 | 144 | 280 |

the increase is to 20 parts in $10^{4}$ at this pressure. Conservatively, we have taken the uncertainties in Z for different temperatures to be fully correlated.

To obtain Z and $\tilde{\mathrm{V}}$ for a mixture of water vapor and oxygen at a particular temperature, a set of virial and pseudovirial coefficients for oxygen up to the sixth (i.e., the coefficient of $\tilde{\mathrm{V}}^{-5}$ ) may be obtained for that temperature from the $Z$ for oxygen and used in eq 5 in the manner and with the justification detailed for water in air (Wylie and Fisher, 1996).
$B_{\text {ow }}$ and $C_{\text {oow }}$ for the Experimental Temperatures. The analysis closely parallels that for water in air. For a given temperature the changes in the calculated $\ln f_{w}$ for increments in the $B_{\text {ow }}$ and $C_{\text {oow }}$ substituted in eq 6 are almost proportional to $p$ and $p^{2}$, respectively. Thus, when approximate values of those coefficients are substituted and the differences in the calculated values of In $f_{w}$ from the experimental values are least-squares-fitted by a quadratic polynomial in $p$, the coefficients found for $p$ and $p^{2}$ give adjustments which should be made to $\mathrm{B}_{\mathrm{ow}}$ and $\mathrm{C}_{\text {oow }}$. Even a single iteration of the procedure is likely to suffice. The constant term obtained in the least-squares fit, which we denote by a, ideally should be zero, but inevitably a small nonzero value is found. This is believed to arise largely from the presence of a small amount of argon in the oxygen for which no allowance has been made in the molecular weight used in eq 9 . The deviations of the individual measurements from the final fits for the three temperatures are plotted in Figure 1.

For each experimental temperature, as a mathematical property of the least-squares fitting process, the random uncertainties in the values found for $\mathrm{B}_{\text {ow }}$ and $\mathrm{C}_{\text {oow }}$ are highly correlated. Indeed, the correlation coefficient for $-\mathrm{B}_{\text {ow }}$ and $\mathrm{C}_{\text {oow }}$ is 0.98 , which we round to unity. The random uncertainties in the values found for different experimental temperatures are, of course, uncorrelated. The systematic uncertainties in $\mathrm{B}_{\mathrm{ow}}$ and $\mathrm{C}_{\text {oow }}$ are the sums (with due regard to sign) of the uncertainties in eq 6 which are proportional to $p$ and $p^{2}$, respectively. The term in $C_{o w w}$ can be resolved without significant residue into a term proportional to $p$ and another proportional to $\mathrm{p}^{2}$, and makes the dominant contributions to these systematic uncertainties. Therefore, the conservative approximation has been made that all the systematic uncertainties are mutually fully correlated (with proper regard to sign). The values found for $\mathrm{a}, \mathrm{B}_{\text {ow }}$, and $\mathrm{C}_{\text {oow }}$ and their uncertainties are given in Table 4.

The value of $a$, which is seen to be approximately the same for each experimental temperature, corresponds to an argon content of the commercial oxygen of $0.4 \%$. That the values derived for $\mathrm{B}_{\text {ow }}$ and $\mathrm{C}_{\text {oow }}$ are independent of a means that they are very insensitive to an error in $p_{w}^{o}$ which is constant at a constant temperature and hence to a constant error in any of the experimental temperatures. They are also very insensitive to an error in the adopted molecular weight of the oxygen used, to the value of $Z_{w}^{0}$, and to a constant fractional error in the measured pressure.
$B_{\text {ow }}$ and $C_{\text {oow }}$ for a Range of Temperature. To interpolate and extrapolate $\mathrm{B}_{\text {ow }}(\mathrm{T})$, we have used a leastsquares method to fit an $L_{-J}(6-12)$ potential to the three experimental values of $\mathrm{B}_{\mathrm{ow}}(\mathrm{T})$ (Table 4). In the notation


Figure 2. Derived interaction second virial coefficient $\mathrm{B}_{\mathrm{ow}}(\mathrm{T})$ and its $3 \sigma$ total-uncertainty limits (broken lines). The deviations of the values derived for the three experimental temperatures (see Tables 4 and 5) are not discernible on this scale.
of Hirschfelder et al. (1964) the result is

$$
\begin{gather*}
\epsilon / \mathrm{k}=132.99 \mathrm{~K} \\
\mathrm{~b}_{\mathrm{o}}=59.35 \mathrm{~cm}^{3} / \mathrm{mol} \tag{15}
\end{gather*}
$$

If an L-J (6-18) or an L-J (6-24) potential is fitted instead, the resulting $B_{o w}(T)$ is insignificantly different.

The total uncertainty in a value of $\mathrm{B}_{\mathrm{ow}}(\mathrm{T})$ cal culated from eq 15 comprises six components, three of which are uncorrelated random components arising from the random uncertainties in the three fitted values, and three are corresponding systematic components which, conservatively, have been regarded as mutually fully correlated.

Values of $\mathrm{B}_{\text {ow }}, \mathrm{T}\left(\mathrm{B}_{\text {ow }} / \mathrm{dT}\right)$, and $\mathrm{T}^{2}\left(\mathrm{~d}^{2} \mathrm{~B}_{\text {ow }} / \mathrm{dT}^{2}\right)$ calculated from eq 15 are given with their $3 \sigma$ total uncertainties in Table 5 for temperatures from -100 to $+200^{\circ} \mathrm{C}$. A plot of $\mathrm{B}_{\text {ow }}(\mathrm{T})$ and its uncertainty limits is given in Figure 2 for temperatures from -50 to $+150^{\circ} \mathrm{C}$. The deviations of the experimental values (Table 4) from those in Table 5 are only $-0.04,+0.06$, and $-0.07 \mathrm{~cm}^{3} / \mathrm{mol}$ at 25,50 , and 75 ${ }^{\circ} \mathrm{C}$, respectively. It is not clear why these differences, like the corresponding differences for water in air, are so small.

Lacking reliable theoretical guidance as to the shape of the $\mathrm{C}_{\text {oow }}(\mathrm{T})$ curve, we can do no better than adopt a weighted linear least-squares fit of the three measured values. This gives

$$
\begin{equation*}
\mathrm{C}_{\mathrm{oow}}(\mathrm{~T}) /\left(\mathrm{cm}^{3} / \mathrm{mol}\right)^{2}=1420-5.1 \mathrm{t} /{ }^{\circ} \mathrm{C} \tag{16}
\end{equation*}
$$

The total uncertainty in a value of $\mathrm{C}_{\text {oow }}(\mathrm{T})$ calculated from eq 16 includes three random and three systematic components analogous to thosefor $\mathrm{B}_{\text {ow }}$, but for $\mathrm{C}_{\text {oow }}$ we have included a fourth (statistically independent) systematic component to allow for our lack of knowledge of the true shape of the curve. This component has been taken to vary smoothly from $100 \mathrm{~mL} / \mathrm{mol}$ at $0^{\circ} \mathrm{C}$ to 40 at 25,20 at 50,30 at 75 , and 60 at 100 .

The relationship of eq 16 to the three experimental values and their $3 \sigma$ total uncertainties may be seen in

Table 5. Values and $3 \sigma$ Uncertainties of $B_{o w}, T\left(d B_{o w} / d T\right)$, and $T^{2}\left(d^{2} B_{o w} / d T^{2}\right)$ for Temperatures from $-\mathbf{1 0 0}$ to $+\mathbf{2 0 0}{ }^{\circ} \mathbf{C}$

| t/ ${ }^{\circ} \mathrm{C}$ | $\mathrm{B}_{\text {ow }} /\left(\mathrm{cm}^{3} / \mathrm{mol}\right)$ | uncertainty/ (cm ${ }^{3} / \mathrm{mol}$ ) | $\begin{aligned} & \hline \mathrm{T}\left(\mathrm{~dB}_{\mathrm{ow}} / \mathrm{dT}\right) / \\ & \left(\mathrm{cm}^{3} / \mathrm{mol}\right) \end{aligned}$ | uncertainty/ ( $\mathrm{cm}^{3} / \mathrm{mol}$ ) | $\begin{gathered} \mathrm{T}^{2}\left(\mathrm{~d}^{2} \mathrm{Bow}_{\mathrm{ow}} / \mathrm{dT}^{2}\right) / \\ \left(\mathrm{cm}^{3} / \mathrm{mol}\right) \end{gathered}$ | uncertainty/ ( $\mathrm{cm}^{3} / \mathrm{mol}$ ) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -100 | -93.8 | 13.6 | 176 | 32 | -432 | 72 |
| -90 | -84.3 | 11.8 | 162 | 30 | -395 | 67 |
| -80 | -76.0 | 10.2 | 150 | 28 | -363 | 63 |
| -70 | -68.7 | 8.9 | 140 | 27 | -335 | 59 |
| -60 | -62.2 | 7.6 | 131 | 25 | -312 | 56 |
| -50 | -56.4 | 6.5 | 123 | 24 | -291 | 53 |
| -40 | -51.2 | 5.5 | 115 | 23 | -273 | 50 |
| -30 | -46.5 | 4.6 | 109 | 21 | -256 | 48 |
| -20 | -42.2 | 3.8 | 103 | 20 | -242 | 45 |
| -10 | -38.3 | 3.1 | 98.1 | 19.5 | -229 | 43 |
| 0 | -34.7 | 2.5 | 93.4 | 18.6 | -217 | 42 |
| 5 | -33.1 | 2.2 | 91.2 | 18.2 | -212 | 41 |
| 10 | -31.44 | 1.90 | 89.0 | 17.8 | -207 | 40 |
| 15 | -29.90 | 1.67 | 87.0 | 17.4 | -202 | 39 |
| 20 | -28.42 | 1.47 | 85.1 | 17.0 | -197 | 38 |
| 25 | -27.00 | 1.32 | 83.2 | 16.7 | -192 | 37 |
| 30 | -25.63 | 1.22 | 81.4 | 16.3 | -188 | 37 |
| 35 | -24.32 | 1.18 | 79.7 | 16.0 | -184 | 36 |
| 40 | -23.05 | 1.20 | 78.0 | 15.7 | -180 | 35 |
| 45 | -21.82 | 1.26 | 76.4 | 15.4 | -176 | 35 |
| 50 | -20.64 | 1.36 | 74.9 | 15.1 | -172 | 34 |
| 55 | -19.51 | 1.49 | 73.4 | 14.8 | -169 | 33 |
| 60 | -18.41 | 1.63 | 72.0 | 14.5 | -166 | 33 |
| 65 | -17.35 | 1.79 | 70.6 | 14.3 | -162 | 32 |
| 70 | -16.32 | 1.95 | 69.2 | 14.0 | -159 | 32 |
| 75 | -15.3 | 2.1 | 68.0 | 13.7 | -156 | 31 |
| 80 | -14.4 | 2.3 | 66.7 | 13.5 | -153 | 31 |
| 85 | -13.4 | 2.4 | 65.5 | 13.3 | -150 | 30 |
| 90 | -12.5 | 2.6 | 64.3 | 13.0 | -148 | 30 |
| 95 | -11.7 | 2.8 | 63.2 | 12.8 | -145 | 29 |
| 100 | -10.8 | 2.9 | 62.1 | 12.6 | -142 | 29 |
| 110 | -9.2 | 3.2 | 60.0 | 12.2 | -138 | 28 |
| 120 | -7.7 | 3.5 | 58.1 | 11.8 | -133 | 27 |
| 130 | -6.3 | 3.8 | 56.2 | 11.4 | -129 | 26 |
| 140 | -4.9 | 4.0 | 54.5 | 11.0 | -125 | 25 |
| 150 | -3.6 | 4.3 | 52.8 | 10.7 | -121 | 25 |
| 160 | -2.4 | 4.5 | 51.2 | 10.4 | -117 | 24 |
| 170 | -1.2 | 4.8 | 49.8 | 10.1 | -114 | 23 |
| 180 | -0.2 | 5.0 | 48.3 | 9.8 | -111 | 23 |
| 190 | 0.9 | 5.2 | 47.0 | 9.5 | -108 | 22 |
| 200 | 1.9 | 5.4 | 45.7 | 9.3 | -105 | 21 |



Figure 3. Values of the interaction third virial coefficient $\mathrm{C}_{\text {oow }}(\mathrm{T})$ derived for the three experimental temperatures, with error bars representing their $3 \sigma$ total uncertainties, and the straight line resulting from a weighted least-squares fit.

Figure 3. Values calculated from the equation are given with their uncertainties in Table 6 for temperatures of $0-100{ }^{\circ} \mathrm{C}$.

## Calculations of $g_{w}$ and $f_{w}$

The vapor concentration enhancement factor $g_{w}(T, p)$ is given ( $a s \ln g_{w}$ ) by the right-hand side of eq 6 with the first term deleted, and may be calculated by substituting data given above, including those derived for $\mathrm{B}_{\mathrm{ow}}(\mathrm{T})$ and $\mathrm{C}_{\text {oow }}(\mathrm{T})$. Every term in the equation for $\ln \mathrm{g}_{\mathrm{w}}$ approaches zero

Table 6. Values and $3 \sigma$ Uncertainties of $\mathrm{C}_{\text {oow }}$ for Temperatures from 0 to $100^{\circ} \mathrm{C}$

| $\mathrm{t} /{ }^{\circ} \mathrm{C}$ | $\mathrm{C}_{\text {oow }} /$ <br> $\left(\mathrm{cm}^{3} / \mathrm{mol}\right)^{2}$ | uncertainty/ <br> $\left(\mathrm{cm}^{3} / \mathrm{mol}\right)^{2}$ | $\mathrm{t} /{ }^{\circ} \mathrm{C}$ | $\mathrm{C}_{\text {oowl }} /$ <br> $\left(\mathrm{cm}^{3} / \mathrm{mol}\right)^{2}$ | uncertainty/ <br> $\left(\mathrm{cm}^{3} / \mathrm{mol}^{2}\right.$ |
| :---: | :---: | :---: | ---: | :---: | :---: |
| 0 | 1420 | 410 | 60 | 1110 | 190 |
| 10 | 1370 | 340 | 70 | 1060 | 230 |
| 20 | 1320 | 270 | 80 | 1010 | 290 |
| 30 | 1270 | 220 | 90 | 960 | 350 |
| 40 | 1220 | 180 | 100 | 910 | 420 |
| 50 | 1170 | 170 |  |  |  |

(and $g_{w}$ approaches unity) as $p$ is decreased to $p_{w}^{0}$. Therefore, there is no question of including the offset a in the equation. The random and systematic uncertainties in $\mathrm{g}_{\mathrm{w}}$ are calculated as the statistical sums of the corresponding uncertainties in the terms, with due regard to the correlations involved. The random uncertainty arises from $B_{o w}$ and $\mathrm{C}_{\text {oow }}$, the six contributions from which being correlated as already described. The systematic uncertainty arises very largely from $\mathrm{B}_{\text {ow }}, \mathrm{C}_{\text {oow, }}$, and $\mathrm{C}_{\text {oww, }}$, and as stated above, its components have been regarded as mutually fully correlated (with regard to sign).

Having obtained $g_{w}$ and its uncertainty, we obtain $f_{w}$ by multiplying by $Z / Z_{w}^{o}$. To obtain the uncertainty of $f_{w}$ from that of $g_{w}$, we statistically delete the uncertainty of $Z$, as its effect when $g_{w}$ is multiplied by $Z / Z_{w}^{0}$ roughly cancels with its effect in the analysis which leads to Table 4, but statistically add in the uncertainty in $Z_{w}^{0}$, which has had no earlier effect because the offset a has been taken equal to zero.

Table 7. Values of $g_{w}$ and $f_{w}$ for Various Temperatures and Pressures ${ }^{a}$

| $\mathrm{t} /{ }^{\circ} \mathrm{C}$ | $\mathrm{p} / \mathrm{MPa}=0.1$ | $\mathrm{p} / \mathrm{MPa}=1$ | $\mathrm{p} / \mathrm{MPa}=2$ | $\mathrm{p} / \mathrm{MPa}=5$ | $\mathrm{p} / \mathrm{MPa}=10$ | $\mathrm{p} / \mathrm{MPa}=15$ |
| ---: | ---: | :--- | :--- | :--- | :--- | :--- |
| 0 | $1.00381,2.2$ | $1.0388,21$ | $1.0789,40$ | $1.2058,87$ | $1.4337,130$ | $1.6660,164$ |
|  | $1.00332,4.6$ | $1.0294,21$ | $1.0592,40$ | $1.1530,87$ | $1.3239,130$ | $1.5089,163$ |
| 20 | $1.00300,1.2$ | $1.0310,11$ | $1.0625,22$ | $1.1602,45$ | $1.3283,58$ | $1.4932,54$ |
|  | $1.00348,4.2$ | $1.0250,12$ | $1.0493,22$ | $1.1251,45$ | $1.2595,57$ | $1.4018,52$ |
| 40 | $1.00231,0.6$ | $1.0249,6$ | $1.0502,11$ | $1.1270,23$ | $1.2555,29$ | $1.3786,27$ |
|  | $1.00423,4.0$ | $1.0224,7$ | $1.0428,12$ | $1.1054,23$ | $1.2149,28$ | $1.3294,23$ |
| 60 | $1.00164,0.4$ | $1.0201,5$ | $1.0406,10$ | $1.1024,21$ | $1.2038,29$ | $1.2996,31$ |
|  | $1.00539,4.0$ | $1.0217,6$ | $1.0391,10$ | $1.0922,20$ | $1.1842,26$ | $1.2801,26$ |
| 80 | $1.00089,0.4$ | $1.0162,7$ | $1.0332,14$ | $1.0839,31$ | $1.1662,44$ | $1.2437,44$ |
|  | $1.00551,4.0$ | $1.0228,8$ | $1.0381,14$ | $1.0843,31$ | $1.1643,42$ | $1.2477,41$ |
| 100 |  | $1.0129,10$ | $1.0272,21$ | $1.0698,48$ | $1.1388,77$ | $1.2039,96$ |
|  |  |  | $1.0255,11$ | $1.0397,21$ | $1.0813,48$ | $1.1530,76$ |

a Upper values, $g_{w}$; Iower values, $f_{w}$. Uncertainties (3 $3 \sigma$ ) are parts in $10^{4}$.


Figure 4. Deviations of the group-average experimental values of $f_{w}$ (values in Table 1 minus the average of the three values of the offset a in Table 4) from the values given by eq 6 with $B_{o w}(T)$ defined by eq 15 or Table 5 and $C_{\text {oow }}(T)$ by eq 16 or Table 6. The broken line represents an unweighted quadratic least-squares fit of the 14 points. Therms deviation of the points from the abscissa is 9.8 parts in $10^{4}$.

Except for this direct effect of an error in $Z_{w}^{0}$ on $f_{w}$, the insensitivity of $\mathrm{B}_{\text {ow }}$ and $\mathrm{C}_{\text {oow }}$ to certain systematic errors, which has been pointed out above, is shared by the calculated values of $g_{w}$ and $f_{w}$.
The group-averaged experimental values of $f_{w}$ (Table 1), with the mean of the three values of the offset a (Table 4) discounted from each, may be compared with values calculated in this way, in effect from eq 6 . The differences of the experimental from the cal culated values are plotted in Figure 4. The rms difference is 9.8 parts in $10^{4}$. A least-
squares quadratic fit of the differences gives the curve shown as a broken line, which practically coincides with the abscissa axis as it should.

A skeletal tabulation of values of $g_{w}$ and $f_{w}$ and their total $3 \sigma$ uncertainties is given in Table 7 for temperatures from 0 to $100^{\circ} \mathrm{C}$ and pressures to 15 MPa .

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